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ON THE CONTINUOUS KOLMOGOROV–WIENER FILTER FOR PREDICTION OF MODELED SMOOTHED HEAVY-TAIL PROCESS

The telecommunication traffic nowadays is treated as a heavy-tail random process. The traffic prediction is an important problem for telecommunications. The paper is devoted to the use of the continuous Kolmogorov–Wiener filter for prediction of modeled smoothed heavy-tail process similar to fractional Gaussian noise, which may describe traffic in a simple model. This process is generated on the basis of the symmetric moving average approach with the use of the exponential smoothing algorithm. In our recent paper we investigated the applicability of the use of both discrete and continuous Kolmogorov–Wiener filter for the prediction of the above-described smoothed heavy-tail modeled data. In particular, it was shown that not only discrete, but also continuous Kolmogorov–Wiener filter may be used to the corresponding prediction, but the accuracy of the discrete filter is slightly higher than the accuracy of the continuous one. In fact, as it was seen from the graphs of the actual and predicted processes, the process predicted on the basis of the continuous filter has some time delay in comparison with the actual process. So, it is logical enough to make a time shift of the predicted process in order to delete the corresponding delay. So, in this paper we propose an enhancement of the corresponding algorithm by using an artificially chosen time shift for the predicted process.

The aim of the work *is to enhance the accuracy of the continuous Kolmogorov–Wiener filter prediction for the smoothed modeled heavy-tail process.*

The methodology *consists in the solving of the Wiener-Hopf integral equation on the basis of the Walsh functions with further use of the time shift for the obtained process.*

The scientific novelty *consists in the enhancement of the corresponding prediction accuracy on the basis of the use of the artificially chosen time shift for the predicted process.*

The conclusions *are as follows. The use of the corresponding time shift allows one to decrease the prediction mean absolute percentage error.*

Key words: *continuous Kolmogorov–Wiener filter, prediction, heavy-tail data, Walsh functions, fractional Gaussian noise, time shift.*

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ДО НЕПЕРЕРВНОГО ФІЛЬТРУ КОЛМОГОРОВА–ВІНЕРА ДЛЯ ПРОГНОЗУВАННЯ ЗМОДЕЛЬОВАНОГО ЗГЛАДЖЕНОГО ПРОЦЕСУ З ВАЖКИМ ХВОСТОМ

В наш час телекомунікаційний трафік вважається випадковим процесом з важким хвостом. Прогнозування трафіку – важлива задача для телекомунікацій. Стаття присвячена застосуванню неперервного фільтра Колмогорова–Вінера для моделювання згладженого процесу з важким хвостом, аналогічного до фрактального гаусівського шуму, що може описати трафік в простій моделі. Цей процес згенеровано на основі підходу симетричного ковзного середнього з використанням алгоритму експоненційного згладжування. В нашій нещодавній статті досліджено можливість застосування як дискретного, так і неперервного фільтру Колмогорова–Вінера для прогнозування вище описаних згладжених даних з важким хвостом. Зокрема, було показано, що не лише дискретний, але і неперервний фільтр Колмогорова–Вінера може бути застосовним до відповідного прогнозування, але точність дискретного фільтру трохи більше за точність неперервного. Фактично, як було видно з графіків фактичного та спрогнозованого процесів, процес, що спрогнозовано на основі неперервного фільтра має певну затримку порівняно з фактичним процесом. Тож, логічним є зробити часовий зсув спрогнозованого процесу для того, щоб позбутись такої затримки. Тож, в цій статті ми пропонуємо покращення відповідного алгоритму на основі використання штучно обраного часового зсуву для спрогнозованого процесу.

***Метою роботи** є покращити точність прогнозування на основі неперервного фільтра Колмогорова–Вінера для згладженого змодельованого процесу з важким хвостом.*

***Методологія** полягає в розв'язанні інтегрального рівняння Вінера–Хопфа на основі функцій Уолша з подальшим використанням часового зсуву для отриманого процесу.*

***Наукова новизна** полягає у покращенні відповідної точності прогнозування на основі використання штучно обраного часового зсуву для спрогнозованого процесу.*

***Висновки** є такими. Використання відповідного часового зсуву дозволяє зменшити середню абсолютну відсоткову помилку прогнозування.*

***Ключові слова:** неперервний фільтр Колмогорова–Вінера, прогнозування, дані з важким хвостом, функції Уолша, фрактальний гаусівський шум, часовий зсув.*

Introduction. The telecommunication traffic prediction is an urgent problem for telecommunications, see the description of problem importance and a review of different traffic prediction methods in (Alizadeh et al, 2020). Our recent papers were devoted to such a simple prediction method as the Kolmogorov–Wiener filter. In particular, in papers (Gorev et al, 2022; Gorev et al, 2023) we generated a heavy-tail process similar to fractional Gaussian noise on the basis of the symmetric moving average approach (Koutsoyiannis, 2002). It should be stressed that nowadays telecommunication traffic in systems with data packet transfer is treated as a heavy-tail process, see (Li, 2022). The applicability of the Kolmogorov–Wiener filter to the prediction of the corresponding process was investigated. In paper (Gorev et al, 2023) it is shown that both discrete and continuous Kolmogorov–Wiener filter may be applied to the corresponding prediction, but the prediction based on the continuous filter leads to a small time delay between the actual and the predicted process. So, in this paper we propose to use an artificially chosen time shift for the predicted process in order to delete such a delay.

Process prediction based on the continuous Kolmogorov–Wiener filter. This section is devoted to the corresponding use of the continuous Kolmogorov–Wiener filter, described in (Gorev et al, 2023). As is known, the filter weight function $h(\tau)$ is a solution of the Wiener–Hopf integral equation

$$\int_0^T h(\tau)R(t-\tau)d\tau = R(t+z) \quad (1)$$

where T is the time interval for which the input data are given, z is the time interval for which the forecast is made, R(t) is the process correlation function, see for example, (Gorev, Gusev, Korniienko, Aleksieiev, 2021; Miller, Childers, 2012). The weight function is sought as a Walsh function truncated series (a partial case of the Galerkin method, see (Polyanin, Manzhirov, 2008)); the description of the corresponding Walsh functions in Walsh numeration is given in (Gorev, Gusev, Korniienko, 2021; Gorev, Gusev, Korniienko, Safarov, 2021). The weight function may be calculated as follows (Gorev et al, 2023):

$$h(\tau) = \sum_{s=0}^{n-1} g_s S_s(\tau),$$

$$\begin{pmatrix} g_0 \\ g_1 \\ \vdots \\ g_{n-1} \end{pmatrix} = \begin{pmatrix} G_{00} & G_{01} & \cdots & G_{0,n-1} \\ G_{10} & G_{11} & \cdots & G_{1,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ G_{n-1,0} & G_{n-1,1} & \cdots & G_{n-1,n-1} \end{pmatrix}^{-1} \cdot \begin{pmatrix} B_0 \\ B_1 \\ \vdots \\ B_{n-1} \end{pmatrix}, \quad (2)$$

here $S_s(\tau)$ are the Walsh functions in the Walsh numeration. The approximation of 256 Walsh functions is used. The prediction is made on the basis of the data given on previous time interval with the length T=1 for the time interval z=10^{-3} forward. The corresponding process contains 10^3 points on the unit time interval, so, in fact, a one-point-forward prediction is considered. The process is generated on the time interval $t \in [0,100]$, so it contains 10^5 points. There are rather many points on the unit time interval, so the process may be treated as a continuous one. The filter output for the centralized process prediction is as follows:

$$yc(T+iz) = \int_0^T h(\tau)\,xc\big(T+(i-1)\cdot z-\tau\big)d\tau \approx$$
$$\approx \sum_{j=0,10^{-3},2\cdot10^{-3},\ldots}^{T-10^{-3}} h(j)\cdot xc\big(T+(i-1)\cdot z-j\big)\cdot 10^{-3} \quad (3)$$

where $xc(t) = x(t) - \langle x \rangle$ is the centralized process and $x(t)$ is the modeled exponentially smoother heavy-tail process generated in (Gorev et al, 2023). The prediction values for the process $x(t)$ are as follows:

$$Y(t) = y(t+\Delta t). \quad (4)$$

The correlation function of the exponentially smoothed heavy-tail process investigated in (Gorev et al, 2023) is as follows:

$$R(t) = \begin{cases} a_1|t|^3 + b_1|t|^2 + c_1|t| + d_1, |t| \in [0, 0.1] \\ a\cdot\left(|t| + 10^{-3}\right)^{-b}, |t| \in (0.1, 1] \end{cases} \quad (5)$$

$a_1 = -369$, $b_1 = 79.0$, $c_1 = -5.89$, $d_1 = 0.220$, $a = 0.0206$, $b = 0.415$,

the numerical values of the coefficients in (5) are rounded off to 3 significant digits. The graphs of the actual and predicted processes for $t \in (1,1.2)$ are as follows, see Fig. 1. The average mean absolute percentage error (MAPE) of such a prediction is equal to 5.8% (Gorev et al, 2023), so the corresponding prediction is rather accurate. However, as can be seen from Fig. 1, the corresponding graphs are rather close, but a small time delay is present between the actual and the predicted signals.

So, the next section is devoted to the use of the time shift which may delete such a delay.

The use of the time shift to the predicted process. The following algorithm of the time shift choice is realized. Let us build a time shifted process as

$$Y(t)=y(t+\Delta t). \quad (6)$$

where $y(t)$ is the process given by (4). Let us consider $Y(t)$ as the predicted process. In such a case the results for the average MAPE are given in Table 1.

Table 1

Average MAPE for different time shifts

Average MAPE over the whole array, %	**Time shift Δt from expression (6)**
5.8	0
3.7	10^{-3}
2.5	$2\cdot10^{-3}$
3.6	$3\cdot10^{-3}$
5.8	$4\cdot10^{-3}$
8.2	$5\cdot10^{-3}$

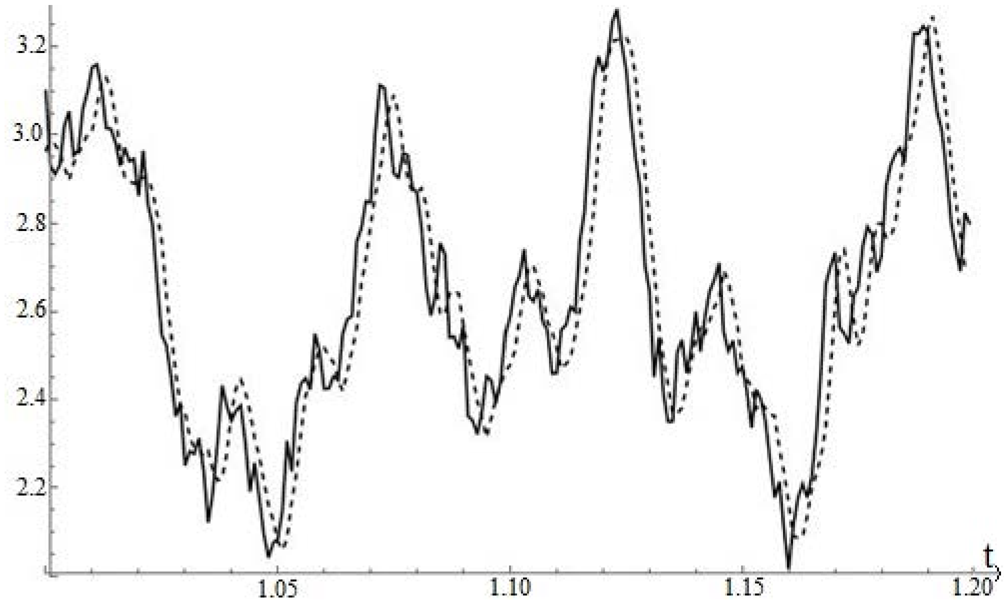

Fig. 1. Comparison of the actual (solid line) and predicted (dotted line) processes (Gorev et al, 2023)

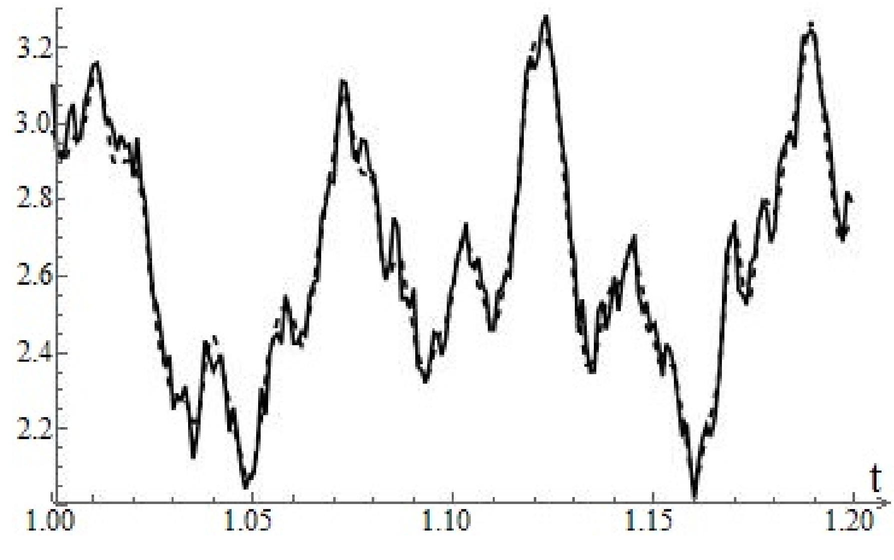

Fig. 2. Comparison of the actual (solid line) and predicted (dotted line) processes after the use of the time shift

The further increment of Δt leads to the increasing of the MAPE. So, as can be seen from Table 1, the optimal time shift is equal to $2{\cdot}10^{-3}$. In such a case the average MAPE is equal to 2.5% rather than 5.8%, so an enhancement of the process prediction accuracy is obtained. The MAPE results in Table 3 are rounded off to two significant digits. The graphical comparison of the actual process $x(t)$ and the predicted process $Y(t)$ for $t\in(1,1.2)$ is given on Fig. 2.

As can be seen from Fig. 2, the use of the time shift makes the predicted process closer to the actual one and increases the prediction accuracy. It should be stressed that the MAPE values in Table 1 are calculated over the whole array rather than only over the time interval $t\in(1,1.2)$.

Conclusions. This paper is devoted to the prediction of the modeled exponentially smoothed heavy-tail process generated in (Gorev et al, 2023). There are rather many process points on the unit time interval, so the process may be treated as a continuous one. The applicability of the continuous Kolmogorov–Wiener filter to the process prediction is shown in (Gorev et al, 2023), however, as can be seen from Fig. 1, the process predicted on the basis of the continuous Kolmogorov–Wiener filter has a time delay in comparison with the actual one. So, in this paper the time shift of the predicted process is proposed in order to delete the above-mentioned delay. It is shown that the use of an optimal time shift allows one to increase the prediction accuracy. So, the proposed prediction algorithm is as follows. First of all one should obtain the prediction based on the Kolmogorov–Wiener filter and then the obtained predicted process should be time shifted in order to increase the process accuracy.

REFERENCES:

1. Alizadeh M., Beheshti M. T. H., Ramezani A., Saadatinezhad H. Network Traffic Forecasting Based on Fixed Telecommunication Data Using Deep Learning. *Proceedings of the 2020 6th Iranian Conference on Signal Processing and Intelligent Systems (ICSPIS)*, 2020. doi: 10.1109/ICSPIS51611.2020.9349573.

2. Gorev V., Gusev A., Korniienko V. The use of the Kolmogorov–Wiener filter for prediction of heavy-tail stationary processes. *CEUR Workshop Proceedings*. 2022. Vol. 3156, p. 150–159. Available at: http://ceur-ws.org/Vol-3156/paper9.pdf.

3. Gorev V., Gusev A., Korniienko V., Shedlovska Y. On the use of the Kolmogorov–Wiener filter for heavy-tail process prediction. *Journal of Cyber Security and Mobility*. 2023. In press.

4. Koutsoyiannis D. The Hurst phenomenon and fractional Gaussian noise made easy. *Hydrological Sciences Journal*. 2002. Vol. 47, p. 573-595. doi: 10.1080/02626660209492961.

5. Li M. Fractal Teletraffic Modeling and Delay Bounds in Computer Communications. Boca Raton: CRC Press, 2022. doi: 10.1201/9781003268802.

6. Gorev V., Gusev A., Korniienko V., Aleksieiev M., Kolmogorov–Wiener Filter Weight Function for Stationary Traffic Forecasting: Polynomial and Trigonometric Solutions. *Lecture Notes in Networks and Systems*. 2021. Vol. 212, p. 111–129. doi: 10.1007/978-3-030-76343-5_7.

7. Miller S. L., Childers D. Probability and Random Processes With Applications to Signal Processing and Communications, 2nd. ed. Amsterdam : Elsevier, Academic Press, 2012. doi: 10.1016/C2010-0-67611-5.

8. Polyanin A. D., Manzhirov A. V. Handbook of integral equations. Second edition. New York: Chapman and Hall, 1144 p, 2008. doi: 10.1201/9781420010558

9. Gorev, V., Gusev A., Korniienko V. Fractional Gaussian Noise Traffic Prediction Based on the Walsh Functions. *CEUR Workshop Proceedings*. 2021. Vol 2853, p. 389–400. Available at: http://ceur-ws.org/Vol-2853/paper35.pdf.

10. Gorev V. N., Gusev A. Y., Korniienko V. I., Safarov A. A. On the Kolmogorov–Wienre filter for random processes with a power-law structure function based on the Walsh functions. *Radio Electronics, Computer Science, Control*. 2021. No. 2, p. 39–47. doi: 10.15588/1607-3274-2021-2-4.