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ELECTRODYNAMIC ALGORITHM FOR CALCULATING AN ANTENNA ARRAY BASED ON AN INTEGRAL REPRESENTATION FOR A COMMON REGION FIELD

The article is devoted to the study of a new approach in the integral equation method. The solution of the electrodynamic problem is carried out based on conditional selection of the common region in the entire area of electromagnetic field definition. In our previous publications, we proposed an integral equation method based on the selection of a penetrating region for solving the problem of electromagnetic wave diffraction on a periodic structure. The legality of using the proposed approach and its equivalence with the previously proposed approach are shown.

In this article, the calculation of an infinite antenna array scanning in the H-plane is carried out. The numerical convergence of the proposed approach for reflection coefficients R_{10} with increasing order of truncation of the system of linear algebraic equations was studied. It was found that the modulus of the reflection coefficient coincides with the exact solution at $M = 1$ and subsequently does not change with the growth of M. While the phase of the reflection coefficient changes with the growth of M and when M >11, the difference between the phase value of the exact solution and the calculated value is less than 1%. That is, for the case of scanning in the H-plane, good convergence of the problem solution was obtained for all scanning angles.

The aim of the work *is to show that for a certain class of problems of applied electrodynamics, it is possible to apply the integral equation method based on the selection of a common region.*

The methodology *consists in the conditional allocation of a common region in the entire area of electromagnetic field determination and the application of the integral equation method.*

The scientific novelty *is that we have shown the correctness of using a new approach based on the selection of the field of the common region for the calculation of periodic structures in the H-plane.*

The conclusions *can be formulated as follows. It is shown that two approaches can be used to calculate the antenna array in the H-plane: the first based on the selection of the integral representation for the full field of the penetrating region, the second based on the selection of the integral representation for the full field of the common region.*

Key words: *integral equation method, penetrating domain, common region, reflection coefficient, numerical convergence.*

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ЕЛЕКТРОДИНАМІЧНИЙ АЛГОРИТМ РОЗРАХУНКУ АНТЕННОЇ РЕШІТКИ НА ОСНОВІ ІНТЕГРАЛЬНОГО ПРЕДСТАВЛЕННЯ ДЛЯ ПОЛЯ СПІЛЬНОЇ ОБЛАСТІ

Статтю присвячено дослідженню нового підходу в методі інтегрального рівняння. Розв'язок електродинамічної задачі проводиться на основі умовного виділення спільної області у всій області визначення електромагнітного поля. В наших попередніх публікаціях пропонувався метод інтегрального рівняння на основі виділення проникливої області для розв'язку задачі дифракції електромагнітної хвилі на періодичній структурі. Показана правомірність застосування запропонованого підходу і його еквівалентність із раніш запропонованим підходом.

В даній статті проведено розрахунок нескінченої антенної решітки, що сканує в Н-площині. Досліджена чисельна збіжність запропонованого підходу для коефіцієнтів відбиття R_{10} при збільшенні порядку усічення системи лінійних алгебраїчних рівнянь. Отримано, що модуль коефіцієнта відбиття співпадає із точним рішенням при $M = 1$ і в подальшому не змінюється з ростом M. В той час як фаза коефіцієнта відбиття змінюється з ростом M і при M >11 різниця між величиною фази точного рішення та обчислювальної величини складає менше 1%. Тобто, для випадку сканування в Н-площині для всіх кутів сканування отримана гарна збіжність рішення задачі.

Мета роботи – *показати, що для певного класу задач прикладної електродинаміки можна застосовувати метод інтегрального рівняння на основі виділення спільної області.*

Методологія *полягає в умовному виділенні спільної області у всій області визначення електромагнітного поля і застосуванні метода інтегрального рівняння.*

Наукова новизна *полягає в тому, що ми показали коректність застосування нового підходу на основі виділення поля спільної області для розрахунків періодичних структур в Н-площині.*

Висновки *можна сформулювати таким чином. Показано, що для розрахунку антенної решітки в Н-площині можна використовувати два підходи: перший на основі виділення інтегрального представлення для повного поля проникливої області, другий на основі виділення інтегрального представлення для повного поля спільної області.*

Ключові слова: *метод інтегрального рівняння, прониклива область, спільна область, коефіцієнт відбиття, чисельна збіжність.*

Introduction. Nowadays, in the field of wireless communications, there is a rapid transition to a higher frequency range of microwaves. Development of a new frequency range requires the creation of new emitting devices. One of the types of modern devices is MASSIVE MIMO and Intelligent Reflecting Surface. The creation of such devices takes place through mathematical modeling. Therefore, there is a need to improve the methods used in mathematical modeling, particularly the integral equation method.

Currently, interest remains in improving the integral equation method for its application to the calculation of various radiating structures (Li X. at al, 2018; Li H. at al, 2018; Morozov, Magro, 2020; Morozov, Magro, 2021). Although the mode matching method can be considered «classical», it is constantly developing and improving depending on the type of devices for which mathematical modeling is performed (Morozov, Magro, 2020; Zheng, Yu, 2007). The emergence of new complex emitting devices leads to the need to consider the value of the dielectric constant of materials located in the aperture of the emitter. Therefore, there are various improvements in the integral equation method (Gnilenko, Magro, 2017; Sun, Zhao, Huang, 2019; Bie, Peng, Jiang, 2021; Magro, Morozov, 2022).

The purpose of the work is to show that for a certain class of problems of applied electrodynamics it is possible to apply the integral equation method based on the selection of a common region. In our previous works, we proposed a new approach in the integral equation method (Magro, Morozov, 2000). It is based on the conditional selection of a penetrating region on the entire electromagnetic field definition area. In this work, an approach in the integral equation method is proposed, which is based on the conditional allocation of a common area in the entire area of electromagnetic field determination.

Electrodynamic algorithm based on an integral representation for a general area field.

Let us consider the infinite a waveguide antenna array consisting of open ends of a waveguide. The antenna array is surrounded by an endless metal screen. In an infinite antenna array, the fields are identical in all periodic cells, except for the phase value, which changes by a constant value in each subsequent element. Therefore, we will consider the field only in one cell, which is located at the origin of coordinates. The cross section of an infinite antenna array of plane-parallel waveguides for the case of scanning in the H-plane (the electric field strength vector is directed along the OY axis), Fig. 1. We conditionally divide the domain of definition of the electromagnetic field into three overlapping regions: 1 is waveguide region extended to infinity $-w/2 \le x \le w/2$; $-\infty \le z \le \infty$; 2 is half-space above a metal screen $-F/2 \le x \le F/2$; $0 \le z \le \infty$; C is common region $-w/2 \le x \le w/2$; $0 \le z \le \infty$.

In the common region, at points $(-w/2, 0)$; $(w/2)$ the field $E_y^0(x,0) = 0$ is equal to zero, therefore the Green's function of this region will be equal to zero at other points $G^0(x,0;x',z') = 0$. Then we can write the following integral representation

$$E_y^0(x,z) = \int_0^{\infty}\left[E_y^0(-w/2,z') \frac{\partial G^0(x,z;x',z')}{\partial x'}\bigg|_{\substack{x'=-w/2 \\ z\prec z'}} - E_y^0(w/2,z') \frac{\partial G^0(x,z;x',z')}{\partial x'}\bigg|_{\substack{x'=w/2 \\ z\prec z'}} \right] dz' + \int_{-w/2}^{w/2} E_y^0(x',0) \frac{\partial G^0(x,z;x',z')}{\partial z'}\bigg|_{\substack{z'=0 \\ z\succ z'}} dx', \quad (1)$$

$$x, x' \in [-w/2, w/2];\ z' \in [0,\infty];\ z = 0.$$

Here G^0 is the Green's function of the common region, which has the following form:

$$G^0(x,z;x',z') = \sum_{q=1}^{\infty} D_q(x) D_q(x') \frac{1}{jwl_q} \begin{cases} e^{-jwl_q z} sh(jwl_q z'), & z \succ z'; \\ e^{-jwl_q z'} sh(jwl_q z), & z \prec z'. \end{cases} \quad (2)$$

We use the following boundary condition:

$$E_y^2(x,z) = E_y^0(x,z), \quad z = 0. \quad (3)$$

Considering these boundary conditions, the Green's function of the common region will take the following values:

$$G^0(x,0;x',z')\big|_{z\prec z'} = 0; \quad \frac{\partial G^0(x,0;x',z')}{\partial x'}\bigg|_{x'=\pm w/2} = 0\,. \quad (4)$$

Considering equations (3) and (4), equation (1) can be rewritten in the following form:

$$E_y^2(x,0) = \int_{-w/2}^{w/2} E_y^1(x',0) \frac{\partial G^0(x,z;x',z')}{\partial z'}\bigg|_{\substack{z'=0 \\ z\succ z'}} dx'\,. \quad (5)$$

We use the following boundary conditions:

$$H_x^1(x,z) = H_x^0(x,z),\ z = 0\,;$$

$$\frac{\partial E_y^1(x,z)}{\partial z}\bigg|_{z=0} = \frac{\partial E_y^0(x,z)}{\partial z}\bigg|_{z=0}\,. \quad (6)$$

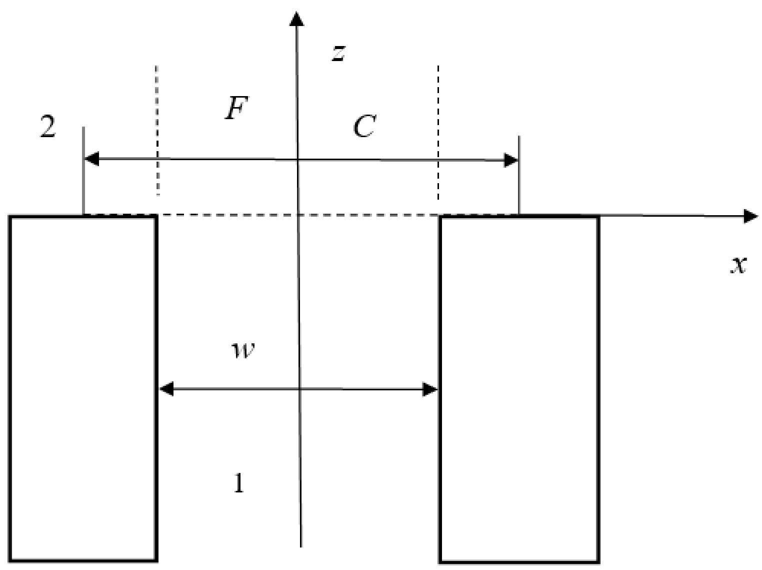

Fig. 1. Single antenna array cell with dedicated areas

Considering boundary conditions (6), equation (1) takes the form:

$$\left.\frac{\partial E_y^1(x,z)}{\partial z}\right|_{z=0^-} = \left.\frac{\partial}{\partial z}\right|_{z=0^-}\int_0^\infty (E_y^2(-w/2,z')\left.\frac{\partial G^0(x,z;x',z')}{\partial x'}\right|_{\substack{x'=-w/2\\ z\prec z'\\ z=0^-}} -$$

$$-E_y^2(w/2,z')\left.\frac{\partial G^0(x,z;x',z')}{\partial x'}\right|_{\substack{x'=w/2\\ z\prec z'\\ z=0^-}})dz' +$$

$$+\left.\frac{\partial}{\partial z}\right|_{z=0^+}\int_{-w/2}^{w/2} E_y^1(x',0)\left.\frac{\partial G^0(x,z;x',z')}{\partial z'}\right|_{\substack{z'=0\\ z\succ z'\\ z=0^+}} dx', \qquad (7)$$

$$z=\begin{cases}0^-, z'\in[0,\infty];\\ 0^+, z'=0.\end{cases}$$

Thus, in equation (7), when differentiating both integrals, the source (integration) points and the observation points do not coincide. That is, the integrand function has no singularities. Therefore, it is possible to introduce differentiation operations under the signs of integrals. As a result, we obtain the following system of equations:

$$E_y^2(x,0)=\int_{-w/2}^{w/2} E_y^1(x',0)\left.\frac{\partial G^0(x,z;x',z')}{\partial z'}\right|_{\substack{z'=0\\ z\succ z'\\ z=0}} dx';$$

$$\left.\frac{\partial E_y^1(x,z)}{\partial z}\right|_{z=0} = \int_0^\infty \left[E_y^2(-w/2,z')\left.\frac{\partial^2 G^0(x,z;x',z')}{\partial z\partial x'}\right|_{\substack{x'=-w/2\\ z\prec z'\\ z=0}} -\right.$$

$$\left.-E_y^2(w/2,z')\left.\frac{\partial^2 G^0(x,z;x',z')}{\partial z\partial x'}\right|_{\substack{x'=w/2\\ z\prec z'\\ z=0}}\right]dz' +$$

$$+\int_{-w/2}^{w/2} E_y^1(x',0)\left.\frac{\partial^2 G^0(x,z;x',z')}{\partial z\partial z'}\right|_{\substack{z'=0\\ z\succ z'\\ z=0}} dx'. \qquad (8)$$

Let us represent the fields in the first and second regions in the form of a Fourier series expansion (Amitay, Galindo, Wu, 1972):

$$E_y^1(x,z)=\sum_{n=1}^{\infty} R_n D_n(x) e^{jwl_n z} + E_{y\,exc}^1(x,z); \qquad (9)$$

$$E_y^2(x,z)=\sum_{n=1}^{\infty} T_m f_m(x) e^{-jfl_n z}, \qquad (10)$$

here $f_m(x)$ is complex orthonormal system of transverse eigenfunctions of radiation space (Amitay, Galindo, Wu, 1972).

Let us substitute expressions (2), (9), (10) into the first equation of system (8). Then we have:

$$\sum_{m=-\infty}^{\infty} T_m f_m(x) = \sum_{n=1}^{\infty}\sum_{q=1}^{\infty} R_n D_q(x)\int_{-w/2}^{w/2} D_n(x')D_q(x')dx' +$$

$$+\sum_{q=1}^{\infty} D_q(x)\int_{-w/2}^{w/2} D_1(x')D_q(x')dx'.$$

As a result, we get the expression:

$$\sum_{m=-\infty}^{\infty} T_m f_m(x) = \sum_{n=1}^{\infty} R_n D_n(x) + D_1(x);\ x\in[-w/2, w/2]. \quad (11)$$

Equation (11) is equivalent to the usual «mode-matching method». Let's multiply the left and right sides of equation (11) by $D_v(x)$ and integrate within $[-w/2, w/2]$.

$$\sum_{m=-\infty}^{\infty} T_m \int_{-w/2}^{w/2} f_m(x)D_v(x)dx =$$

$$=\sum_{n=1}^{\infty} R_n \int_{-w/2}^{w/2} D_n(x)D_v(x)dx + \int_{-w/2}^{w/2} D_1(x)D_v(x); \quad (12)$$

$$R_v + \sum_{m=-\infty}^{\infty} T_m\,(-C_{vm}) = \delta_{1v}.$$

Let us substitute expressions (2), (9), (10) into the second equation of system (8). Then we have the following expression

$$\sum_{n=1}^{\infty} R_n\, jwl_n\, D_n(x) - jwl_1\, D_1(x) =$$

$$=\sum_{m=-\infty}^{\infty}\sum_{q=1}^{\infty} T_m\left[f_m(-w/2)\left.\frac{dD_q(x')}{dx'}\right|_{x'=-w/2} - f_m(w/2)\left.\frac{dD_q(x')}{dx'}\right|_{x'=w/2}\right].$$

$$D_v(x)\int_0^\infty e^{-j(fl_m+wl_q)z'} +$$

$$+\sum_{n=1}^{\infty}\sum_{q=1}^{\infty} R_n(-j)wl_q\, D_q(x)\int_{-w/2}^{w/2} D_n(x')D_q(x')dx +$$

$$+\sum_{q=1}^{\infty}(-j)wl_q\, D_q(x)\int_{-w/2}^{w/2} D_1(x')D_q(x')dx.$$

After transformations we obtain the following equation

$$\sum_{n=1}^{\infty} R_n\, jwl_n\, D_n(x) = \sum_{m=-\infty}^{\infty}\sum_{q=1}^{\infty} T_m\, A''_{qm}\, B_{qm}\, D_q(x) +$$

$$+\sum_{n=1}^{\infty} R_n\,(-j)wl_n\, D_n(x). \qquad (13)$$

Let's multiply the left and right sides of equation (13) by $D_v(x)$ and integrate within $[-w/2, w/2]$

$$\sum_{n=1}^{\infty} R_n\, jwl_n \int_{-w/2}^{w/2} D_n(x)D_v(x)dx =$$

$$=\sum_{m=-\infty}^{\infty}\sum_{q=1}^{\infty} T_m\, A''_{qm}\, B_{qm}\int_{-w/2}^{w/2} D_q(x)D_v(x)dx +$$

$$+\sum_{n=1}^{\infty} R_n\,(-j)wl_n \int_{-w/2}^{w/2} D_n(x)D_v(x)dx.$$

As a result, we get the following equation:

$$-2\,jwl_v\, R_v + \sum_{=-\infty}^{\infty} T_m\, A''_{qm}\, B_{vm} = 0. \qquad (14)$$

Thus, equations (13) and (14) form the following system of equations:

$$-2\,j\,wl_v\,R_v + \sum_{=-\infty}^{\infty} T_m\,A''_{qm}\,B_{vm} = 0\,; \qquad (15)$$

$$R_v + \sum_{m=-\infty}^{\infty} T_m\,(-C_{vm}) = \delta_{1v}\,.$$

Let us exclude the unknown coefficient R_v from system (15)

$$R_v = \sum_{m=-\infty}^{\infty} T_m\,C_{vm} - \delta_{1v}\,; \qquad (16)$$

$$\sum_{m=-\infty}^{\infty} (C_{vm}\,2\,j\,wl_v - A''_{vm}\,B_{vm})T_m = 2\,j\,wl_m\,\delta_{1v}\,.$$

Let's multiply equation (16) by $(\frac{-1}{2\,j\,wl_v})$ and consider that A_{vm} has the following form:

$$A_{vm} = \frac{1}{2\,j\,wl_v}A''_{vm}\,.$$

We finally get the following expression:

$$\sum_{m=-\infty}^{\infty} (C_{vm} - A_{vm}\,B_{vm})\,T_m = \delta_{1v}. \qquad (17)$$

The resulting system of equations (17) is equivalent to the system of equations obtained by the penetrating domain method (Morozov, Magro, 2000).

As a test to verify the correctness of the proposed algorithm, we examine a linear antenna array that scans in the H-plane. In table 1 shows the results of a study of the numerical convergence of the solution for the reflection coefficient R_{10} with increasing order of truncation of a system of linear algebraic equations ($F/\lambda = 0.5714$, $t = 0$). In this case, symmetrical truncation was carried out, i.e. the equality $M^+ = M^-$, $M = M^+ + M^- + 1$ was satisfied. The data in Table 1 correspond to the case of waveguides with infinitely thin walls ($t = \frac{F - w}{w} = 0$) and the value $F/\lambda = 0.5714$. Calculations were carried out for scanning angles $\theta = 2.87^0;10^0;40^0;51^0$. It was found that at scanning angles $\theta \le 40^0$, good convergence of the problem solution was obtained. For scanning angles $\theta \ge 51^0$, good convergence is also observed, but it is necessary to consider the larger number $M \ge 31$. Here, the difference between the phase value of the exact solution and the calculated phase value with an error of less than 2% is achieved at $M \ge 31$. Thus, for the case of scanning in the H-plane, good numerical convergence of the solution to the problem was obtained.

Conclusions. An electrodynamic algorithm was built for the first time, which allows for mathematical modeling of a certain class of applied electrodynamics problems. In the work, it is proposed to distinguish a general area from the entire area of electromagnetic field definition. The convergence of the proposed method was studied. It is shown that two approaches can be used to calculate the

Table 1

Investigation of numerical convergence of the solution for different scanning angles (Magro, Morozov, 2023)

M	$\|R_{10}\|$	**phase, deg.**	$\|R_{10}\|$	**phase, deg.**
	$\theta = 2.87^0$		$\theta = 40^0$	
1	0.347	180.0	0.226	180.0
3	0.347	165.2	0.226	123.7
5	0.347	161.7	0.226	118.8
7	0.347	160.1	0.226	116.9
9	0.347	159.2	0.226	115.9
11	0.347	158.6	0.226	115.2
13	0.347	158.2	0.226	114.8
15	0.347	157.9	0.226	114.4
17	0.347	157.7	0.226	114.2
19	0.347	157.5	0.226	114.0
21	0.347	157.4	0.226	113.8
	$\theta = 10^0$		$\theta = 51^0$	
1	0.341	180.0	0.130	180.0
3	0.341	163.7	0.046	36.2
5	0.341	160.1	0.460	30.2
7	0.341	158.5	0.460	28.6
9	0.341	157.6	0.460	27.5
11	0.341	157.0	0.460	26.8
13	0.341	156.6	0.460	26.4
15	0.341	156.3	0.460	26.0
17	0.341	156.1	0.460	25.8
19	0.341	155.9	0.460	25.6
21	0.341	155.7	0.460	25.4

antenna array in the H-plane: the first based on the selection of the integral representation for the full field of the penetrating region, the second based on the selection of the integral representation for the full field of the general region. The proposed approach in the integral equation method allows to solve the radiation problems of modern antenna arrays.

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