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POLYNOMIAL SOLUTIONS FOR THE KOLMOGOROV–WIENER PREDICTION OF MODELED SMOOTHED HEAVY-TAIL PROCESS

Nowadays telecommunication traffic in systems with data packet transfer is considered as a heavy-tail random process. In a couple of rather simple models traffic is considered to be stationary one. In our recent papers we generated modeled heavy-tail data, which is based on the smoothing of the fractional Gaussian noise. In particular, the applicability of the continuous Kolmogorov–Wiener filter to the prediction of such data was investigated, the corresponding Wiener–Hopf integral equation was solved on the basis of the truncated Walsh function expansion. However, a question occurs – may another truncated orthogonal function expansion be applied to the problem under consideration? So, the corresponding investigation may be an actual question. In our recent papers we investigated theoretical fundamentals of the Kolmogorov–Wiener filter construction for different models, in particular, on the basis of the truncated polynomial expansion method and on the basis of the truncated trigonometric Fourier series expansion method. In this paper we restrict ourselves to the investigation of the applicability of the truncated polynomial expansion method to the problem under consideration, the corresponding method is based on the Chebyshev polynomials of the first kind. The applicability of another polynomial or trigonometric expansions to the problem under consideration may be discussed in other papers.

The aim of the work is to investigate the applicability of the Galerkin method based on the Chebyshev polynomials of the first kind to the Kolmogorov–Wiener prediction of smoothed heavy-tail data.

The methodology consists in the solving of the Wiener–Hopf integral equation on the basis of the truncated polynomial expansion method which is based on the Chebyshev polynomials of the first kind.

The scientific novelty consists in the proof of the fact that the Galerkin method based on the Chebyshev polynomials of the first kind may be applied to the Kolmogorov–Wiener prediction of smoothed heavy-tail data.

The conclusions are as follows. The truncated polynomial expansion method based on the Chebyshev polynomials of the first kind may give reliable results in the framework of the Kolmogorov–Wiener prediction of smoothed heavy-tail data.

Key words: continuous Kolmogorov–Wiener filter, prediction, smoothed heavy-tail data, Chebyshev polynomials of the first kind.

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ПОЛІНОМІАЛЬНІ РОЗВ'ЯЗКИ ДЛЯ ПРОГНОЗУВАННЯ КОЛМОГОРОВА–ВІНЕРА МОДЕЛЬНИХ ДАНИХ ЗГЛАДЖЕНОГО ПРОЦЕСУ З ВАЖКИМ ХВОСТОМ

На сьогоднішній день телекомунікаційний трафік в системах з пакетною передачею даних розглядається як випадковий процес з важким хвостом. У низці досить простих моделей трафік вважається стаціонарним. У наших нещодавніх роботах ми згенерували модельні дані з важким хвостом, які базують-

ся на згладжуванні фрактального гаусівського шуму. Зокрема, досліджено застосовність неперервного фільтра Колмогорова–Вінера до прогнозування таких даних, розв'язано відповідне інтегральне рівняння Вінера–Хопфа на основі обірваного розвинення по функціям Уолша. Але виникає питання – чи можна застосувати до задачі, що розглядається, інше обірване розвинення за ортогональними функціями? Отже, відповідне дослідження може бути актуальним. У наших останніх роботах ми досліджували теоретичні основи побудови фільтра Колмогорова–Вінера для різних моделей, зокрема, на основі методу обірваного поліноміального розвинення та на основі методу обірваного розвинення в тригонометричний ряд Фур'є. В цій роботі ми обмежимося дослідженням застосовності до розглянутої задачі методу обірваного розвинення по поліномам, який базується на поліномах Чебишова першого роду. Застосовність інших поліноміальних або тригонометричних розкладів до задачі, що розглядається, може бути обговорена в інших статтях.

Метою роботи є дослідження застосовності методу Галеркіна на основі поліномів Чебишова першого роду до прогнозування Колмогорова–Вінера згладжених даних з важким хвостом.

Методологія полягає в розв'язуванні інтегрального рівняння Вінера–Хопфа на основі методу обірваного розвинення по поліномам Чебишова першого роду.

Наукова новизна полягає в доведенні того факту, що метод Галеркіна на основі поліномів Чебишова першого роду може бути застосований до прогнозування Колмогорова–Вінера згладжених даних з важким хвостом.

Висновки є такими. Метод обірваного розвинення за поліномами Чебишева першого роду може дати хороші результати в рамках прогнозування Колмогорова–Вінера згладжених даних з важким хвостом.

Ключові слова: неперервний фільтр Колмогорова–Вінера, прогнозування, згладжені дані з важким хвостом, поліноми Чебишова першого роду.

Introduction. Nowadays telecommunication traffic in systems with packet data transfer is considered as a self-similar heavy-tail random process, see, for example, (Kozlovskiy et al, 2023). The problem of telecommunication traffic prediction is actual for telecommunications. In a couple of models telecommunication traffic may be treated as stationary one, so the problem of the prediction of stationary heavy-tail data may be considered. In our recent papers we generated heavy-tail smoothed modeled data which is based on the smoothing of fractional Gaussian noise data. The Kolmogorov–Wiener prediction of such data was investigated in (Gorev et al, 2022a; Gorev et al, 2023a; Gorev et al, 2023b). In particular, it was shown that continuous Kolmogorov–Wiener filter prediction may be applied to the prediction of the above-mentioned modeled data, the corresponding Wiener–Hopf equation was treated on the basis of the Galerkin method (Polyanin & Manzhirrov, 2008) based on the Walsh functions. However, a question occurs – may the Galerkin method for the problem under consideration be based on another truncated orthogonal function expansion? In paper (Gorev et al, 2021a) a review of the theoretical fundamentals of the Kolmogorov–Wiener filter prediction based on the Galerkin method is given, different polynomial systems and trigonometric Fourier series were used as the basis of the Galerkin method. In papers (Gorev et al, 2020; Gorev et al, 2021b; Gorev et al, 2022b) the Chebyshev polynomials of the first kind were used on the framework of the corresponding Galerkin methods for different models. So, the aim of this paper is to investigate the applicability of the Galerkin

method based on the Chebyshev polynomials of the first kind to the Kolmogorov–Wiener prediction of smoothed heavy-tail data generated in (Gorev et al, 2023a).

Continuous Kolmogorov–Wiener prediction based on the Chebyshev polynomials of the first kind. In this section the Wiener–Hopf integral equation is solved with the help of the Galerkin method based on the Chebyshev polynomials of the first kind. The Wiener–Hopf integral equation is as follows, see, for example, (Gorev et al, 2023b):

$$\int_0^T h(\tau)R(t-\tau)d\tau = R(t+z) \quad (1)$$

where $h(\tau)$ is the unknown weight function, T is the time interval on which the observed data are given, z is the time interval on which the prediction is made, $R(t)$ is the process correlation function. In the framework of the Galerkin method, the unknown weight function is sought as

$$h(t) = \sum_{s=0}^{n-1} g_s S_s(t) \quad (2)$$

where $S_s(t)$ are the functions orthogonal on $t \in (0, T)$. In this paper we use the Chebyshev polynomials of the first kind:

$$S_s(t) = T_s\left(\frac{2t}{T} - 1\right),$$

$$T_s(x) = \sum_{k=0}^{\lfloor s/2 \rfloor} C_s^{2k} (x^2 - 1)^k x^{n-2k}, \quad (3)$$

$\lfloor s/2 \rfloor$ is the integer part of $s/2$, the functions $S_s(t)$ are orthogonal on $t \in (0, T)$, see (Gorev et al, 2020). The coefficients g_s in (2) are given by the following expression (Gorev et al, 2023b):

$$h(\tau) = \sum_{s=0}^{n-1} g_s S_s(\tau),$$

$$\begin{pmatrix} g_0 \\ g_1 \\ \vdots \\ g_{n-1} \end{pmatrix} = \begin{pmatrix} G_{00} & G_{01} & \cdots & G_{0,n-1} \\ G_{10} & G_{11} & \cdots & G_{1,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ G_{n-1,0} & G_{n-1,1} & \cdots & G_{n-1,n-1} \end{pmatrix}^{-1} \begin{pmatrix} B_0 \\ B_1 \\ \vdots \\ B_{n-1} \end{pmatrix},$$

$$G_{ij} = \int_0^T \int_0^T S_i(\tau) S_j(t) R(t-\tau) dt d\tau,$$

$$B_i = \int_0^T S_i(t) R(t+z) dt. \quad (4)$$

The correlation function of the smoothed heavy-tail data generated in (Gorev et al, 2023a) is as follows:

$$R(t) = \begin{cases} a_1 |t|^3 + b_1 |t|^2 + c_1 |t| + d_1, & |t| \in [0, 0.1] \\ a \cdot (|t| + 10^{-3})^{-b}, & |t| \in (0.1, 1] \end{cases},$$

$$a_1 = -369, \quad b_1 = 79.0, \quad c_1 = -5.89, \quad d_1 = 0.220,$$

$$a = 0.0206, \quad b = 0.415, \quad (5)$$

the numerical values of the coefficients in (5) are rounded off to 3 significant digits.

In (Gorev et al, 2023a), the problem under consideration was solved on the basis of the Walsh function expansion, the average mean absolute percentage error (MAPE) for the approximation of 256 Walsh functions is equal to 5.8%. Such MAPE may be reduced on the basis of the time shift proposed in (Gorev et al, 2023b).

In this paper the problem is investigated on the basis of the Chebyshev polynomials of the first kind (3). The integral brackets G_{ij} from (4) are calculated on the basis of the NIntegrate operation built in the Wolfram Mathematica, the integrals are previously treated as follows:

$$\int_0^T \int_0^T S_i(\tau) S_j(t) R(t-\tau) dt d\tau = \{x = t - \tau, y = t + \tau\} =$$

$$= \frac{1}{2} \iint_{\Omega} S_i\left(\frac{x+y}{2}\right) S_j\left(\frac{x-y}{2}\right) R(x) dx dy =$$

$$= \frac{1}{2} \int_{-T}^0 dx R(x) \int_{-x}^{x+2T} dy S_i\left(\frac{x+y}{2}\right) S_j\left(\frac{x-y}{2}\right) +$$

$$+ \frac{1}{2} \int_0^T dx R(x) \int_x^{2T-x} dy S_i\left(\frac{x+y}{2}\right) S_j\left(\frac{x-y}{2}\right), \quad (6)$$

the square region Ω is restricted by the lines $y = x$, $y = -x$, $y = 2T - x$, $y = 2T + x$.

The free terms B_i are estimated on the basis of the method of trapezoids:

$$B_i = \int_0^T S_i(t) R(t+z) dt \approx$$

$$\approx \frac{1}{2} \cdot \frac{1}{10^3} \cdot \sum_{j=0}^{10^3-1} \left(S_i\left(\frac{j}{10^3}\right) R\left(\frac{j}{10^3} + z\right) + S_i\left(\frac{j+1}{10^3}\right) R\left(\frac{j+1}{10^3} + z\right) \right), \quad (7)$$

because the NIntegrate operation may not give reliable results for B_i if the number of polynomials is rather large, see a similar situation described in (Gorev et al, 2022c).

The calculation of the process prediction is similar to that described in (Gorev et al, 2023a; Gorev et al, 2023b). The following average MAPE results are obtained for different numbers of polynomials, see Table 1.

The MAPE values in Table 1 are rounded off to 3 significant digits. The MAPE begins to increase if $n \geq 25$, in our opinion such a fact may be explained by increasing of the algorithm calculation errors for large numbers of polynomials.

As can be seen, the MAPE for rather large n is not very high, so the Galerkin method based on the Chebyshev polynomials of the first kind may be applied to the problem under consideration. However, the method based on the Walsh functions may lead to the MAPE equal to 5.8%, see (Gorev et al, 2023a; Gorev et al, 2023b). After the use of the time shift which maximally decreases the MAPE, the results are 4.97% for the method based on the Chebyshev polynomials of the first kind and 2.5% (Gorev et al, 2023b) for the method based on the Walsh functions. So, the method based on the Walsh functions gives better results for the problem under consideration.

Conclusions. This paper is devoted to the investigation of the applicability of the Galerkin method based on the Chebyshev polynomials of the first kind to the Kolmogorov–Wiener prediction of modeled smoothed heavy-tail data generated in (Gorev et al, 2023a). It is shown that the method based on the Chebyshev polynomials of the first kind may be applied to the problem under consideration, however, the Walsh function expansion method, used in (Gorev et al, 2023a; Gorev et al, 2023b) gives better results than the method based on the Chebyshev polynomials. In our opinion, such a fact may be explained as follows. The Chebyshev polynomials with rather large numbers contain the products of rather large values and rather large exponents of the argument, so the numerical calculation based on the Chebyshev polynomials may meet difficulties. The Walsh functions do not have such drawbacks, moreover, the Walsh

Table 1

Average MAPE for different numbers of polynomials

n	MAPE, %	n	MAPE, %	n	MAPE, %
1	23.3	9	14.4	17	9.19
2	21.9	10	13.3	18	8.83
3	20.6	11	12.5	19	8.51
4	19.5	12	11.9	20	8.22
5	18.5	13	11.2	21	7.98
6	17.4	14	10.7	22	7.82
7	16.4	15	10.1	23	7.73
8	15.4	16	9.65	24	7.71

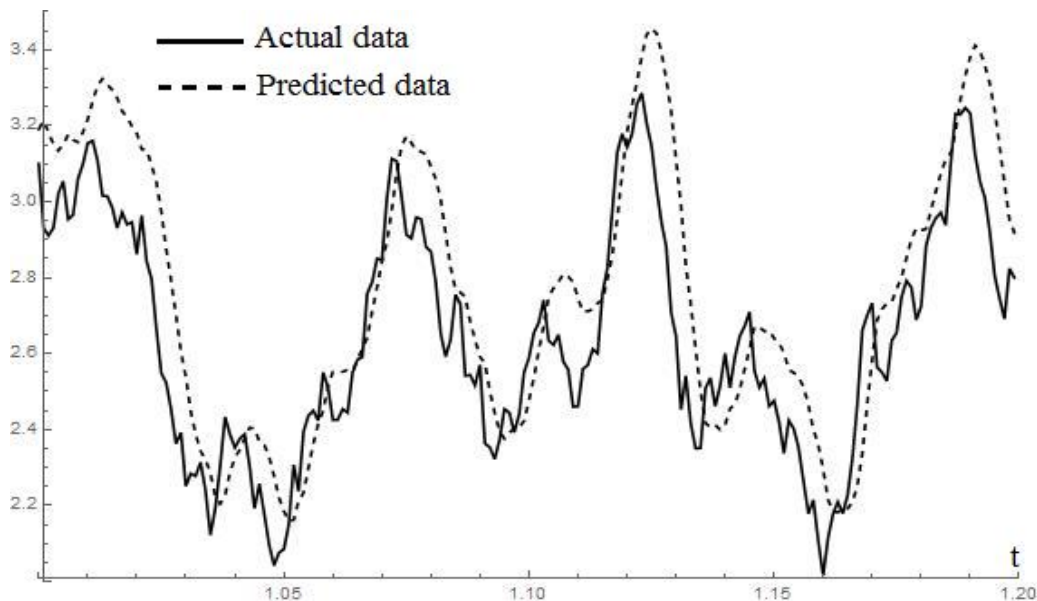


Fig. 1. Comparison of the actual and predicted data for the approximation of 24 polynomials

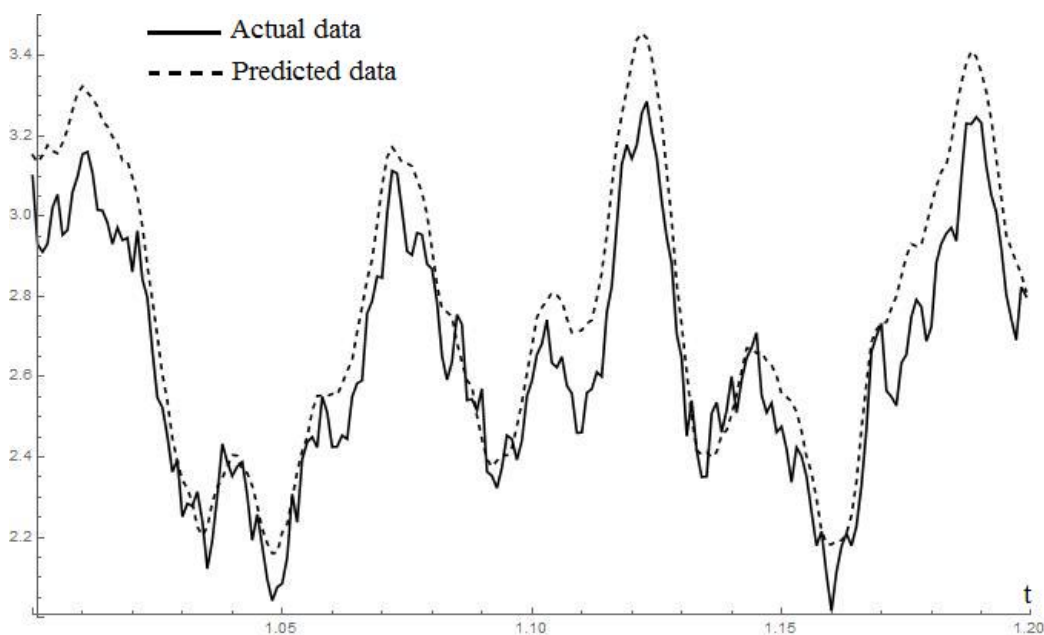


Fig. 2. Comparison of the actual and predicted data for the approximation of 24 polynomials after the use of the time shift equal to $3 \cdot 10^{-3}$

functions are piecewise constant ones, so they can easily be treated. Maybe, future enhancement of the correlation function (5) or enhancement of the method used in (7) may enhance the prediction based on the Chebyshev polynomials.

The paper results may be applied not only to the telecommunications, but also to other fields of knowledge where heavy-tail processes may take place. For example, they may be applied to the

monitoring of soil and climatic parameters of agro-technical objects.

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