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ON THE ACCURACY OF SOME APPROXIMATIONS FOR THE KOLMOGOROV–WIENER FILTER WEIGHT FUNCTION FOR POWER–LAW STRUCTURE FUNCTION PROCESSES

The paper is devoted to the investigation of the accuracy of some polynomial approximations for the Kolmogorov–Wiener filter weight function. The corresponding filter is applied to the prediction of stationary random processes with a power-law structure function. In our previous investigations the Kolmogorov–Wiener filter weight function was obtained on the basis of the truncated polynomial expansion method based on the Chebyshev polynomials of the first kind. It was obtained that some approximations lead to good results; however, some approximations (i.e. the approximations of 9–15 polynomials) fail. The corresponding conclusion was made on the basis of the evaluation of the integrals with the help of the NIntergate function built in the Wolfram Mathematica package. In this paper the corresponding integrals are evaluated on the basis of the rectangle method, the method of trapezoids, and the Simpson method. It is shown that, in contrast to the previous investigations, the approximations of 9–15 polynomials do lead to good results.

***The aim of the work** is to show that, in contrast to the results of previous investigations, the considered polynomial approximations are rather accurate.*

***The methodology** consists in the use of the rectangle method, the method of trapezoids, and the Simpson method for the calculation of the left-hand side of the Wiener–Hopf integral equation for the obtained weight function.*

***The scientific novelty** consists in showing the validity of some polynomial approximations based on the Chebyshev polynomials of the first kind in the framework of the problem under consideration.*

***The conclusions** are as follows. In contrast to the results of previous investigations, it is shown that the approximations of 9–15 Chebyshev polynomials of the first kind for the Kolmogorov–Wiener filter weight function for the prediction of stationary random processes with a power-law structure function are rather accurate.*

***Key words:** Kolmogorov–Wiener filter weight function, Chebyshev polynomials of the first kind, numerical integration methods, power-law structure function prosess.*

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ДО ТОЧНОСТІ ДЕЯКИХ НАБЛИЖЕНЬ ДЛЯ ВАГОВОЇ ФУНКЦІЇ ФІЛЬТРА КОЛМОГОРОВА–ВІНЕРА ДЛЯ ПРОЦЕСІВ ЗІ СТЕПЕНЕВОЮ СТРУКТУРНОЮ ФУНКЦІЄЮ

Статтю присвячено дослідженню точності деяких поліноміальних наближень для вагової функції фільтра Колмогорова–Вінера. Відповідний фільтр застосовано до прогнозування стаціонарних випадкових процесів зі степеневою структурною функцією. В наших попередніх дослідженнях вагова функція фільтра Колмогорова–Вінера була отримана на основі методу обірваних поліноміальних розвинень, що базується на поліномах Чебишова першого роду. Було отримано, що деякі наближення призводять до хороших результатів, однак деякі наближення – ні (а саме, наближення 9–15 поліномів). Відповідний висновок було зроблено на основі обчислення інтегралів за допомогою функції NIntergate, яка є вбудованою в пакет Wolfram Mathematica. В цій статті відповідні інтеграли обчислено на основі методу прямокутників, методу трапецій та методу Сімпсона, і показано, що на відміну від результатів попередніх досліджень, наближення 9–15 поліномів дійсно призводять до хороших результатів.

Мета роботи – *показати, що на відміну від результатів попередніх досліджень, поліноміальні наближення, що розглядаються, є достатньо точними.*

Методологія *полягає у використання методу прямокутників, методу трапецій та методу Сімпсона для обчислення лівої частини інтегрального рівняння Вінера–Хопфа для отриманої вагової функції.*

Наукова новизна *полягає в тому, що ми показали застосовність деяких поліноміальних наближень основаних на поліномах Чебишова першого роду до задачі, що розглядається.*

Висновки *можна сформулювати таким чином. На відміну від результатів минулих досліджень, показано, що наближення 9–15 поліномів Чебишова першого роду для вагової функції Колмогорова–Вінера для прогнозування стаціонарних процесів зі степеневою структурною функцією є достатньо точними.*

Ключові слова: *вагова функція фільтра Колмогорова–Вінера, поліноми Чебишова першого роду, методи числового інтегрування, процес зі степеневою структурною функцією.*

Introduction. The telecommunication traffic prediction is an important problem for telecommunications. In particular, it is an urgent problem for cyber security because a cyber-attack may be detected by comparing the predicted traffic with the actual one, see (Katris, Daskalaki, 2015; Brugner, 2017; Iqbal, Zahid, Habib, John; 2019).

There are a plenty of different rather sophisticated approaches to traffic prediction; for example, see the review in (Katris, Daskalaki, 2015; Brugner, 2017; Iqbal, Zahid, Habib, John; 2019). However, if the telecommunication traffic is a stationary one, such sophisticated approaches as ARIMA, neural networks and wavelet transforms may not be needed. As is known, such a simple approach as the Kolmogorov–Wiener filter may be used for the prediction of stationary random processes (Diniz, 2020). Our recent research was devoted to obtaining the Kolmogorov–Wiener filter weight function for the prediction of stationary telecommunication traffic in the framework of different models. In particular, some papers were devoted to the investigation of theoretical fundamentals of the Kolmogorov–Winer filter construction for the prediction of traffic in a model where it is considered as a random process with a power-law structure function.

The corresponding investigations on the basis of the truncated polynomial expansion method are described in papers [5–8] (Gorev, Gusev, Korniienko, 2019; Gorev, Gusev, Korniienko, 2019; Gorev, Gusev, Korniienko, 2020; Gorev, Gusev, Korniienko, 2021). In paper (Gorev, Gusev, Korniienko, 2019) polynomials orthogonal without weight are used; in papers (Gorev, Gusev, Korniienko, 2019; Gorev, Gusev, Korniienko, 2020) the Chebyshev polynomials of the second and first kind are used, respectively. In paper (Gorev, Gusev, Korniienko, 2021) the MAEs (mean absolute errors) of the considered polynomial approximations are given. In those papers the calculations were made on the basis of the Wolfram Mathematica package, and, in particular, the left-hand side of the Wiener–Hopf integral equation was calculated with the help of the NIntergate function built in the Wolfram Mathematica package. In was shown that the approximations mostly give reliable results; however, the approximations of 9–15 polynomials fail.

The aim of this work is to demonstrate that, in contrast to our previous results, the above-mentioned approximations of 9–15 polynomials are indeed valid. For these purpose we calculate the left-hand side of the Wiener–Hopf integral equation on the basis of the widely known methods of numerical integration rather than the NIntergate function. We use such methods as the rectangle method, the method of trapezoids, and the Simpson method. This paper is devoted only to the investigation of the Chebyshev polynomials of the first kind. The preliminary results devoted to the Chebyshev polynomials of the second kind are published in (Gorev, 2021).

Wiener–Hopf integral equation and the truncated polynomial expansion method. In this paper we investigate the use of the Kolmogorov–Wiener filter to the prediction of a stationary random process with a power-law structure function. The correlation function of such a process is as follows, see, for example, (Gorev, Gusev, Korniienko, 2019):

$$R(t) = \sigma^2 - \frac{\alpha}{2}|t|^{2H} \quad (1)$$

where σ is the process variance, H is the Hurst exponent, and α is a proportionality coefficient between $|t|^{2H}$ and the structure function. The traffic is considered as a random continuous stationary process; such a consideration may be valid in the case of a large amount of data. The Kolmogorov–Winer filter weight function obeys the following Wiener–Hopf integral equation:

$$\int_0^T h(\tau) R(t-\tau) d\tau = R(t+z) \quad (2)$$

where T is the time interval for which the input data are given, and z is the time interval for which the forecast is made. In papers (Gorev, Gusev, Korniienko, 2019; Gorev, Gusev, Korniienko, 2019; Gorev, Gusev, Korniienko, 2020; Gorev, Gusev, Korniienko, 2021; Gorev, 2021) the following numerical values for these parameters were investigated:

$$\sigma = 1.2,\ H = 0.8,\ \alpha = 3\cdot 10^{-3},\ T = 100,\ z = 3. \quad (3)$$

The explicit analytical solution of the integral equation (2) meets difficulties, so the solution of (2) may be found on the basis of the truncated polynomial expansion method (see, for example, (Gorev, Gusev, Korniienko, 2020)). In the case of the Chebyshev polynomials of the first kind, the method is as follows. The unknown weight function in the n–polynomial approximation is sought as a truncated series (Gorev, Gusev, Korniienko, 2020):

$$h(\tau) = \sum_{s=0}^{n-1} g_s T_s\left(\frac{2\tau}{T} - 1\right), \quad (4)$$

where g_s are the unknown coefficients, and $T_s(x)$ are the Chebyshev polynomials of the first kind. On the basis of (1) – (4) one can obtain the following system of linear equations in g_s:

$$\sum_{s=0}^{n-1} g_s \int_0^T\int_0^T T_k\left(\frac{2t}{T}-1\right) T_s\left(\frac{2\tau}{T}-1\right) R(t-\tau) d\tau dt = \\ = \int_0^T T_k\left(\frac{2t}{T}-1\right) R(t+z) dt \quad (5)$$

which after the evaluation of the corresponding integrals may be solved via the matrix method. The accuracy of the n–polynomial approximation may be checked by comparing the left-hand side and the right-hand side of the integral equation (2) and calculating the MAE:

$$\text{MAE} = \frac{1}{T}\int_0^T \left|\text{Left}(t) - \text{Right}(t)\right|,\ \text{Right}(t) = R(t+z),$$

$$\text{Left}(t) = \int_0^T h(\tau) R(t - \tau) d\tau \qquad (6)$$

where $h(\tau)$ is given by expression (4), and the coefficients g_s are the solution of (5).

In our previous investigations (Gorev, Gusev, Korniienko, 2020; Gorev, Gusev, Korniienko, 2021) we calculated the function $\text{Left}(t)$ in (6) via the NIntergate function built in the Wolfram Mathematica package. The results were as follows: the approximations of rather high numbers of polynomials lead to good results, except for the 9–15 polynomial approximations, which failed. In the framework of this paper we calculate $\text{Left}(t)$ via the rectangle method, the method of trapezoids, and the Simpson method.

The calculation of the left-hand side of the Wiener–Hopf integral equation on the basis of widely known numerical integration methods. In the framework of the rectangle method (the midpoint rule) the function $\text{Left}(t)$ is calculated on the basis of (6) as follows:

$$\text{Left}(t) = \int_0^T h(\tau) R(t - \tau) d\tau \approx \approx \sum_{i=1}^{N} h\left(\frac{\tau_{i-1} + \tau_i}{2}\right) R\left(t - \frac{\tau_{i-1} + \tau_i}{2}\right) \cdot \Delta\tau \qquad (7)$$

where

$$\Delta\tau = \frac{T}{N}, \ \tau_j = \frac{jT}{N}, \qquad (8)$$

N is the number of partition points.

In the framework of the method of trapezoids the function $\text{Left}(t)$ is calculated as

$$\text{Left}(t) \approx \frac{1}{2} \sum_{i=1}^{N} \left(h(\tau_{i-1}) R(t - \tau_{i-1}) + h(\tau_i) R(t - \tau_i) \right) \cdot \Delta\tau . \quad (9)$$

In the framework of the Simpson method the function $\text{Left}(t)$ is as follows:

$$\text{Left}(t) \approx \frac{1}{6} \sum_{i=1}^{N} \Big(h(\tau_{i-1}) R(t - \tau_{i-1}) + +4h\left(\frac{\tau_{i-1} + \tau_i}{2}\right) R\left(t - \frac{\tau_{i-1} + \tau_i}{2}\right) + h(\tau_i) R(t - \tau_i) \Big) \Delta\tau . \quad (10)$$

The value $N = 10^3$ is chosen. The MAE is estimated as

$$\text{Left}(t) = \frac{1}{T} \int_0^T \left| \text{Left}(t) - \text{Right}(t) \right| \approx \approx \frac{1}{N_1 + 1} \sum_{j=0}^{N_1} \left| \text{Left}(\tau_j) - \text{Right}(\tau_j) \right|, \qquad (11)$$

where the value $N_1 = 10^3$ is chosen. The numerical results are given in Table 1.

So, as can be seen, for all the con,sidered methods the approximations of 9–15 polynomials lead to small values of MAE (i.e., to a good agreement between the left-hand side and the right-hand side of the Wiener–Hopf integral equation). The results significantly differ from the corresponding results for the NIntegrate function obtained in (Gorev, Gusev, Korniienko, 2020; Gorev, Gusev,

Table 1

The MAE results for different methods and the comparison with the results of the NIntegrate function (Gorev, Gusev, Korniienko, 2021)

n	MAE, rectangle method	MAE, method of trapezoids	MAE, Simpson method	MAE, NIntegrate function [8]
1	$6{,}4 \cdot 10^{-1}$	$6{,}4 \cdot 10^{-1}$	$6{,}4 \cdot 10^{-1}$	$6{,}4 \cdot 10^{-1}$
2	$2{,}8 \cdot 10^{-2}$	$2{,}8 \cdot 10^{-2}$	$2{,}8 \cdot 10^{-2}$	$2{,}8 \cdot 10^{-2}$
3	$7{,}9 \cdot 10^{-2}$	$7{,}9 \cdot 10^{-2}$	$7{,}9 \cdot 10^{-2}$	$7{,}9 \cdot 10^{-2}$
4	$7{,}7 \cdot 10^{-2}$	$7{,}7 \cdot 10^{-2}$	$7{,}7 \cdot 10^{-2}$	$7{,}7 \cdot 10^{-2}$
5	$8{,}9 \cdot 10^{-3}$	$8{,}9 \cdot 10^{-3}$	$8{,}9 \cdot 10^{-3}$	$8{,}8 \cdot 10^{-3}$
6	$8{,}1 \cdot 10^{-3}$	$8{,}1 \cdot 10^{-3}$	$8{,}1 \cdot 10^{-3}$	$8{,}1 \cdot 10^{-3}$
7	$2{,}8 \cdot 10^{-3}$	$2{,}8 \cdot 10^{-3}$	$2{,}8 \cdot 10^{-3}$	$2{,}8 \cdot 10^{-3}$
8	$2{,}5 \cdot 10^{-3}$	$2{,}5 \cdot 10^{-3}$	$2{,}5 \cdot 10^{-3}$	$2{,}5 \cdot 10^{-3}$
9	$1{,}3 \cdot 10^{-3}$	$1{,}3 \cdot 10^{-3}$	$1{,}3 \cdot 10^{-3}$	$1{,}7 \cdot 10^{2}$
10	$1{,}2 \cdot 10^{-3}$	$1{,}2 \cdot 10^{-3}$	$1{,}2 \cdot 10^{-3}$	$2{,}3 \cdot 10^{2}$
11	$7{,}6 \cdot 10^{-4}$	$7{,}9 \cdot 10^{-4}$	$7{,}5 \cdot 10^{-4}$	$6{,}3 \cdot 10^{3}$
12	$6{,}9 \cdot 10^{-4}$	$7{,}7 \cdot 10^{-4}$	$6{,}7 \cdot 10^{-4}$	$4{,}3 \cdot 10^{4}$
13	$7{,}8 \cdot 10^{-4}$	$5{,}3 \cdot 10^{-4}$	$4{,}7 \cdot 10^{-4}$	$5{,}3 \cdot 10^{5}$
14	$5{,}6 \cdot 10^{-4}$	$1{,}0 \cdot 10^{-3}$	$4{,}2 \cdot 10^{-4}$	$6{,}3 \cdot 10^{6}$
15	$6{,}8 \cdot 10^{-4}$	$1{,}3 \cdot 10^{-3}$	$3{,}1 \cdot 10^{-4}$	$3{,}3 \cdot 10^{7}$
16	$8{,}6 \cdot 10^{-4}$	$1{,}7 \cdot 10^{-3}$	$2{,}9 \cdot 10^{-4}$	$2{,}9 \cdot 10^{-4}$
17	$1{,}1 \cdot 10^{-3}$	$2{,}2 \cdot 10^{-3}$	$2{,}2 \cdot 10^{-4}$	$2{,}7 \cdot 10^{-4}$
18	$1{,}4 \cdot 10^{-3}$	$2{,}7 \cdot 10^{-3}$	$2{,}0 \cdot 10^{-4}$	$1{,}1 \cdot 10^{-3}$

Korniienko, 2021). So, one can conclude that, in the framework of the truncated polynomial expansion method based on the Chebyshev polynomials of the first kind, the NIntegrate function may not be applicable to the calculation of the left-hand side of the Wiener–Hopf integral equation, and the approximations of 9–15 polynomials are indeed accurate enough. It should be stressed that the same situation is observed in (Gorev, 2021) for the Chebyshev polynomials of the second kind.

Conslusions. We investigate the theoretical fundamentals of the Kolmogorov–Wiener filter construction for the prediction of stationary random processes with a power-law structure function. Such an investigation may be applied to the telecommunication traffic prediction, which may be important for cyber security.

The unknown weight function is sought as a truncated series in the Chebyshev polynomials of the first kind. The numerical parameters (3) are investigated. The approximations of rather high numbers of polynomials are accurate enough. It is shown that, in contrast to the results of the previous investigations, the approximations of 9–15 polynomials are rather accurate, and the NIntegrate function, which is built in the Wolfram Mathematica package, may not be applicable for the calculation of the left-hand side of the Wiener–Hopf integral equation for the corresponding approximations.

The results of the paper may be taken into account for the prediction of telecommunication traffic in the power-law structure function model, which may be useful for the detection of cyber-attacks by comparing the predicted traffic with the actual one.

The plan for the future is to show the validity of the corresponding polynomial approximations in the framework of the use of polynomials orthogonal without weight.

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